Building up Spacetime with Quantum Entanglement

Mark van Raamsdonk 2010

https://arxiv.org/abs/1005.3035

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Abstract

Emergence of classically connected spacetimes is intimately related to the quantum entanglement of degrees of freedom in a non-perturbative description of quantum gravity.

Disentangling the degrees of freedom associated with two regions of spacetime results in these **regions pulling apart and pinching off from each other** in a way that can be quantified by standard measures of entanglement.

Background Outline

QM side

Entanglement

Density matrices

Von Neumann entropy

Conformal Field Theory (CFT)

Gravity side

AdS spacetime

AdS/CFT correspondence

Black hole entropy

Ryu–Takayanagi formula

Entanglement — Bell State

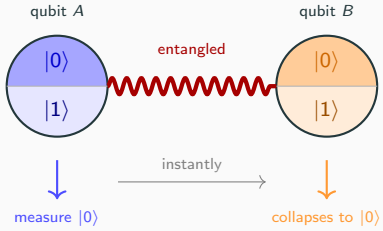
- **Entanglement**

$$|\Psi\rangle \in \mathcal{H} = \mathcal{H}_A \otimes \mathcal{H}_B$$
$$|\Psi\rangle \neq |\psi_A\rangle \otimes |\psi_B\rangle$$

- **Example: Bell state**

$$|\Phi^+\rangle = \frac{1}{\sqrt{2}}\big(|00\rangle + |11\rangle\big)$$

- Neither qubit has a definite state - Maximal Superposition



Entanglement

- **Density matrices**
  - An equivalent formalism for QM
  - Better for working in *mixed* states

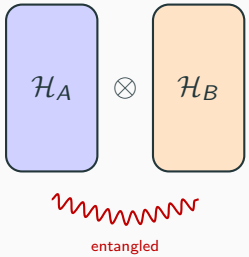
$$\rho_A = \mathrm{tr}_B \, |\Psi\rangle\langle\Psi|$$

- **Von Neumann entropy**
  - Measures *how* entangled two states are

$$S = -\mathrm{tr}(\rho \ln \rho)$$

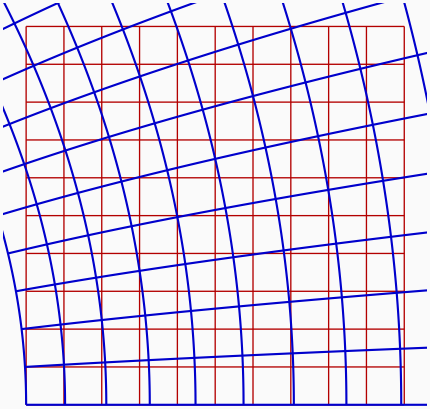
  vanishes iff the state is *separable*

- **Example: Bell State**
  - Entanglement Entropy
    $S = \ln(\dim \mathcal{H}) = \ln 2$



Conformal Field Theory

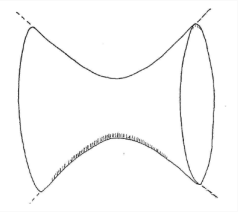
- **Conformal Field Theory (CFT)**
  - A CFT is a quantum field theory that is **invariant** under conformal transformation
  - Conformal transformations preserve angles, but not lengths
  - This extra symmetry lets us avoid perturbation theory



Anti de-Sitter Spacetime

- **Anti de-Sitter Spacetime (AdS)**
  - Derived from Einstein's Field Equations as a **vacuum solution** where the cosmological constant, Λ, is assumed to be negative
  - In reality, theorists work in *asymptotic* Anti de-Sitter spacetime
  - Hyperbolic Spacetime - Land of String Theory

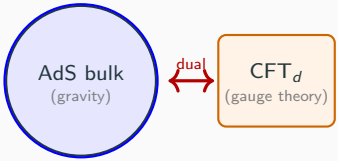
A static slice of AdS - Escher Tessellation



1+1 dimensional AdS

The AdS/CFT Correspondence

- **Maldacena Conjecture (1997):**
  - **The following two theories are mathematically equivalent:**
  - Type IIB strings on $\mathrm{AdS}_5 \times S^5$
  - $\mathcal{N}{=}4$ SYM on $\mathbb{R}^{1,3}$ (Minkowski space)
- Gravity lives in the bulk in $(d{+}1)$-dimensional AdS $\equiv$ d-dimensional CFT on its boundary.

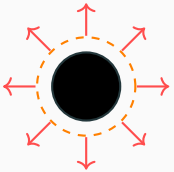


Black Hole Entropy

- **Bekenstein–Hawking Entropy:**

$$S_{\mathrm{BH}} = \frac{A}{4G}$$

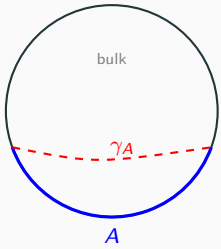
- Entropy scales with *area*, not volume
- First hint towards the holographic principle



Ryu–Takayanagi Entropy

- **Generalization of the Bekenstein–Hawking Entropy**
- The entanglement entropy of a boundary region A is given by the *area* of the shortest surface γ_A in the bulk that shares the same boundary as A:

$$S(A) = \frac{\mathrm{Area}(\gamma_A)}{4G_N}$$



- **Key idea:** how entangled the boundary degrees of freedom are tells you about the *geometry* of the bulk
- **More entanglement means more connected spacetime.**

Two CFT Setup & Bulk Dual

- Take two non-interacting copies of a CFT. Their combined state is:

$$|\psi(\beta)\rangle = \sum_i e^{-\beta E_i/2} |E_i\rangle \otimes |E_i\rangle$$

- No entanglement $\implies$ two *disconnected* spacetimes.
- Entangle them $\implies$ a single **connected** spacetime (eternal AdS black hole).

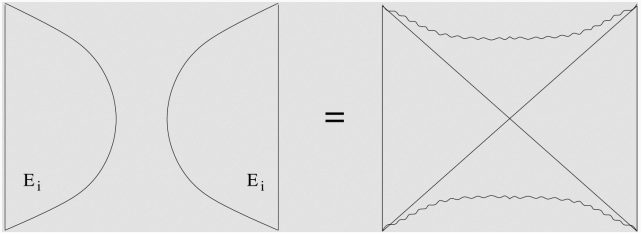


Figure 1 from van Raamsdonk (2010)

van Raamsdonk's Core Argument

- Start with a CFT (dual to pure AdS). Split the boundary into regions A and B.
- Now *decrease* the entanglement between A and B. According to Ryu–Takayanagi:
  - The minimal surface area **shrinks**.
  - The geodesic distance through the bulk goes to **infinity**.
- The two regions **pinch off and pull apart**.
- **"Entanglement is the glue that holds spacetime together."**

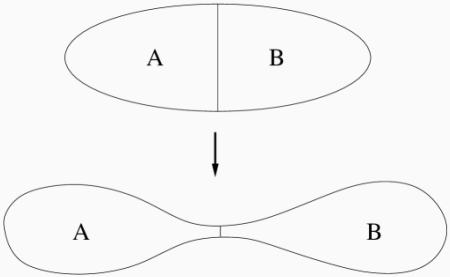


Figure 4 from van Raamsdonk (2010)

Summary & ER=EPR

- **Van Raamsdonk (2010)** argues from AdS/CFT and Ryu–Takayanagi alone that entanglement builds spacetime.
- Takeaways:
  1. Spacetime connectivity emerges from entanglement.
  2. RT entropy is the quantitative link.
  3. Disentangle boundary $\Rightarrow$ disconnect bulk.
- **Later:** Maldacena–Susskind (2013) sharpen this into **EPR = ER**. In essence: Every entangled pair is dual to an Einstein–Rosen bridge (**wormhole**).